störend aus, so daß die Geschwindigkeit erst in größerer Entfernung von der Entladungsstrecke gemessen werden kann. Andererseits ist der Nachweis dieser Vorionisation ein sehr interessantes Nebenergebnis. Verschiedene Autoren[4] haben die Existenz einer solchen Vorionisation zur Erklärung ihrer Resultate angenommen. Experimentelle Ergebnisse liegen noch kaum vor. Auch das Abklingen des Plasmas läßt sich mit Mikrowellen deutlich sichtbar machen, während Trommelkamera und Multiplier in diesem Fall als Diagnostikmittel versagen.

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[4] E. A. McLean, C. E. Faneuff, A. C. Kolb u. H. R. Griem, Phys. Fluids 3, 843 [1960].

A Direct Method of Calculating the Optimum Operating Conditions of Isotope Separation Columns

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The separative power of an infinitesimal counter current element is expressed in terms of radial parameters. It is shown that the separative power is independent of the value of the radial product stream; arbitrarily chosing this to be zero (i. e. "balanced diffusion") makes it possible to calculate the optimum radial isotope velocities and the axial profile directly from the diffusion and continuity equations.

As an illustration of the method, the optimum axial flow profiles and maximum separative powers of gas centrifuges and thermal diffusion columns are calculated, and compared with known results. In the case of the gas centrifuge, the effect that axial back diffusion has on the optimum profile and maximum separative power, is calculated. The variation of the separative power with different axial profiles is determined for the thermal diffusion column. It appears that the natural convection induced axial profile, which is used presently in thermal diffusion columns, gives a small fraction of the maximum separative power. Calculations indicate that a column with a static separative region (and boundary flows) will approach the maximum separative power and will be well suited for measurements of collision parameters.

The isotope separative mechanisms are capable of producing only small changes in enrichment per stage; the elementary separative effect can, however, be considerably enhanced if use is made of a column. In a column, a large number of stages are effectively connected in series by counter current (axial) isotope streams, which move at right angles to the separative effect (radial). The one stream is progressively enriched in the light isotope during the traverse through the column, while the other stream is enriched in the heavy isotope.

The concept of "separative power" is introduced in order to evaluate the isotope separation capability of columns. The separative power of a column is a function of the number of stages in a cascade which are replaced by the column. It is desirable in the design of columns to be able to determine the conditions which make the separative power of the column a maximum.

In the method currently used[1, 2], the differential equation which describes the operation of the column, is brought into the same form as the "fundamental equation of isotope separation", describing the operation of cascades. The optimum operating conditions of the column are then derived within this framework.

The optimization method discussed in this report considers a column of unit height. The radial currents and isotope concentration gradients (which optimise the separative power in the column) are established, and then axial flows are introduced to maintain them. This methods of optimization not

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* On attachment from the South African Atomic Energy Board, Pretoria.

[1] K. P. Cohen, The Theory of Isotope Separation as Applied to the Large Scale Production of U^{235}, McGraw Hill, New York 1951.

[2] K. P. Cohen, Int. Symposium on Isotope Separation, North-Holland Publishing Company, Amsterdam 1957.

only seems to be simpler to use in practice (than the previously described method), but also retains a far clearer picture of the physical mechanisms involved.

1. Theory

The separative power $\mathrm{d}U$ per unit volume in a centrifuge has been given by COHEN[1] in his equation (6.21). Generalized for any type of column, the expression is

$$\mathrm{d}U = \frac{\boldsymbol{J}_1 \cdot \operatorname{grad} N}{N^2(1-N)^2} \text{ moles/cm}^3 \text{ sec.} \tag{1}$$

where $N =$ molar fraction of component 1, $\boldsymbol{J}_1 = c\,N(\boldsymbol{v}_1 - \boldsymbol{v})$, $\boldsymbol{v} = N\,\boldsymbol{v}_1 + (1-N)\,\boldsymbol{v}_2$, $c =$ concentration, $\boldsymbol{v} =$ velocity.

If axial back diffusion is neglected, it follows from (1) that the separative power $\delta U'$ of a column of length l and inner and outer radius r_0 and r_c is

$$\delta U' = \int_{r_0}^{r_c} \frac{(\tau_{1r} - N\,P_r)}{N^2(1-N)^2} \frac{\partial N}{\partial r}\,\mathrm{d}r \text{ moles/sec.} \tag{2 a}$$

where τ_{1z} and τ_{1r} are the components of the transport of component 1, and P_z and P_r are the components of the transport of the mixture ("product stream") in axial and radial directions respectively. Since a stationary state is considered, and since matter is conserved, it follows from (2 a) that

$$\delta U' = \int_{0}^{l} \frac{(\tau_{1z} - N\,P_z)}{N^2(1-N)^2} \frac{\partial N}{\partial z}\,\mathrm{d}z \text{ moles/sec.} \tag{2 b}$$

In isotope separation columns, the basic separative mechanisms operate in the radial direction: the centrifugal field in the gas centrifuge, and the temperature gradient in the thermal diffusion column act radially. The basic diffusion equations for the separative processes are thus also expressed in terms of radial currents and concentrations. It was found that the calculation of optimum axial flow profiles, and the ensuing maximum separative powers, is considerably facilitated if equation (2 a) is used in preference to (2 b). The following procedure of evaluation was adopted.

Because for our argument the length of the column is of no importance we restrict ourselves to a column of unit height.

For a binary mixture, τ_{1r} and P_r are given by:

$$\tau_{1r} = 2\,\pi\,N\,p_{\mathrm{g}}\,r\,v_1/R\,T\,, \tag{3}$$

$$P_r = 2\,\pi[N\,v_1 + (1-N)\,v_2]\,p_{\mathrm{g}}\,r/R\,T \tag{4}$$

where v_1 and v_2 are the radial velocities of components 1 and 2, respectively, (cm/sec), p_{g} is the gas pressure, N is the mol. fraction of component 1 in the gas, R is the gas constant, T is the gas temperature.

In the various isotope separation processes, the radial isotope concentration gradients are generally given by an equation of the form:

$$\partial N/\partial r = A - B(v_1 - v_2) \tag{5}$$

where A is the maximum concentration gradient, which occurs when v_1 and v_2 are zero, B is the reduction of the gradient caused by the isotope velocities.

Substitution of equations (5), (4) and (3) in (2 a) gives:

$$\delta U' = 2\,\pi \int_{r_0}^{r_c} \frac{p_{\mathrm{g}}\,r}{R\,T\,N(1-N)} \left[\frac{A}{B} - \frac{1}{B}\frac{\partial N}{\partial r}\right] \frac{\partial N}{\partial r}\,\mathrm{d}r\,. \tag{6}$$

The maximum separative power occurs when the radial concentration gradient has its optimum value throughout the separative region. Differentiation of equation (6) with respect to the gradient, and equating to zero for the maximum gives:

$$(\partial N/\partial r)_{\text{opt.}} = \tfrac{1}{2}\,A\,. \tag{7}$$

For equations (7) and (6), the maximum separative power is obtaitned as:

$$\delta U'_{\text{max.}} = 2\,\pi \int_{r_0}^{r_c} \frac{p_{\mathrm{g}}\,r}{R\,T\,N(1-N)} \left(\frac{A}{2}\right)^2 \frac{1}{B}\,\mathrm{d}r\,. \tag{8}$$

Substitution of equation (7) in (5) gives the optimum radial velocities as:

$$(v_1 - v_2)_{\text{opt.}} = A/2\,B\,. \tag{9}$$

From equation (8) it is seen that the maximum separative power is not dependent on the value of P_r, so that its value can be chosen arbitrarily. In particular, if P_r is zero ("balanced diffusion") the optimum value of v_1 can be calculated immediately from equations (9) and (4) as:

$$(v_1)_{\text{opt.}} = A(1-N)/2\,B\,. \tag{10}$$

Balanced diffusion operation is attractive in practice since the axial circulation velocity and axial pressure remain constant.

The axial velocity distribution in the separative region, and at the boundaries, is calculated by considering equation (10) in conjunction with the continuity equations. For balanced diffusion in a cylin-

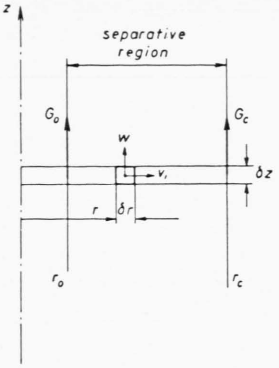

Fig. 1. Flow velocities in column.

drical column with a separative region between radii r_0 and r_c (as shown in Fig. 1), the continuity equations are:

$$w\frac{\partial N}{\partial z} + \frac{v_1 N}{r} + \frac{T}{p_g}\frac{\partial}{\partial r}\left(\frac{p_g v_1 N}{T}\right) = 0\,, \qquad (11)$$

$$G_0 = -\frac{2\pi r_0 N_0 p_{g0} v_{10}}{RT(\partial N/\partial z)}\,, \qquad (12)$$

$$G_c = +\frac{2\pi r_c N_c p_{gc} v_{1c}}{RT(\partial N/\partial z)} \qquad (13)$$

where w is the axial gas velocity at radius r in the separative region (cm/sec), G_0 and G_c are the gas flows at the boundaries of the separative region (g·moles/sec), N_0, N_c, v_{10} and v_{1c} are the mol. fractions and isotope velocities of component 1 at r_0 and r_c, respectively.

Axial back diffusion was neglected in the derivation of equations (11), (12) and (13).

The optimum flow profile is calculated by substituting equations (10) and (7), and an equation for the pressure or temperature distribution in the column, into equation (11). The maximum separative power is calculated by substituting the equations for the temperature and pressure distribution in equation (8).

To illustrate the procedure, the optimum flow profiles and maximum separative powers of counter current gas centrifuges and thermal diffusion columns are determined, and compared with known results. The changes caused by axial back diffusion on the profile and separative power of the gas centrifuge are also considered. For the thermal diffusion column, the separative power is calculated for profiles other than the optimum to determine whether the optimum profile must be exactly reproduced in practice.

Calculations similar to these were initially performed for mass diffusion columns; since no published data on its optimum operation are available, these calculations will be reported when results of an experimental investigation (presently being conducted) are available.

2. Application to gas centrifuge

2.1 *Diffusion equation*

The general diffusion equation of a binary mixture can be put in the form [3]:

$$\frac{N(1-N)(v_1-v_2)}{D_{12}} = -\left\{\frac{\partial N}{\partial r} - N(1-N)(M_1-M_2)\frac{\omega^2 r}{RT} + k_T\frac{1}{T}\frac{\partial T}{\partial r}\right\} \qquad (14)$$

where D_{12} is the coefficient of the diffusion (cm^2/sec), ω is the rate of the rotation (radians/sec), M_1 and M_2 are the masses of the isotopes 1 and 2, respectively, k_T is the thermal diffusion ratio, $k_T = N(1-N)\,H(M_1-M_2)/(M_1+M_2)$, and H is the function containing the collision parameters.

2.2 *Optimum profile in the centrifuge*

For isothermal operation in the gas centrifuge, equation (14) indicates that the values of A and B in equation (6) are:

$$A_c = N(1-N)(M_1-M_2)\,\omega^2 r/RT, \qquad B_c = N(1-N)/D_{12}\,, \qquad (15),\ (16)$$

The optimum radial concentration gradient is obtained from equations (15) and (7) as:

$$(\partial N/\partial r)_{\text{opt.}} = \tfrac{1}{2} N(1-N)(M_1-M_2)\,\omega^2 r/RT\,. \qquad (17)$$

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[3] S. Chapman and T. G. Cowling, The Mathematical Theory of Nonuniform Gases, 2nd edition, Cambridge University Press, Cambridge 1952, p. 244.

For balanced diffusion the optimum radial velocity is obtained from equations (16), (15) and (10) as:

$$v_{1\,\mathrm{opt.}} = D_{12}(1-N)\,(M_1-M_2)\,\omega^2\,r/2\,R\,T\,. \tag{18}$$

The pressure distribution in the centrifuge is given by:

$$\partial p_g/\partial r = \varrho\,\omega^2\,r\,, \qquad \varrho = [M_1\,N + (1-N)\,M_2]\,p_g/R\,T = M\,p_g/R\,T \qquad (19),\ (20)$$

where ϱ is the density of the gas (g/cm^3), M is the average mass of the isotope molecules.

For operation at constant temperature, the product of the diffusion coefficient and the density is a constant, i. e.:

$$D = \varrho\,D_{12} \approx \text{constant}. \tag{21}$$

The optimum profile can now be calculated by substituting equations (21), (19), (18) and (17) in equation (11). This gives:

$$-w\,\frac{\partial N}{\partial z} = \frac{2\,v_1\,N}{r}\left[1 + \frac{1-2\,N}{2}\,(M_1-M_2)\,\frac{\omega^2\,r^2}{4\,R\,T}\right] \tag{22}$$

and

$$\varrho\,w = -\left(\frac{D}{(\partial N/\partial z)}\,(M_1-M_2)\,N(1-N)\,\frac{\omega^2}{R\,T}\left[1 + (M_1-M_2)\,\frac{1-2\,N}{2}\,\frac{\omega^2\,r^2}{4\,R\,T}\right].\right. \tag{23}$$

The last term in equation (23) is generally small in comparison with 1, so that the product $\varrho\,w$ is not a function of the radius. The optimum flow profile thus requires that the axial mass flow is constant at all radial positions. This result has also been derived by Los [4].

The maximum separative power in the absence of axial back diffusion is calculated by substituting equations (19), (16) and (15) in equation (8):

$$\delta U' = \frac{\pi\,D}{2\,M}\left[\frac{M_1-M_2}{2\,R\,T}\,\omega^2\right]^2 (r_c{}^4 - r_0{}^4) \tag{23 a}$$

which is similar to the result given by Groth [5].

2.3 *Optimum flow profile with axial back diffusion*

It is desirable to ascertain whether axial back diffusion influences the optimum flow profile. The equation for axial diffusion in the separative region is:

$$-\frac{\partial N}{\partial z} = \frac{N(1-N)}{D_{12}}\,(w_1 - w_2) \tag{24}$$

where w_1 and w_2 are the axial velocities of components 1 and 2 (in the separative region).

The molecular flow continuity equations for components 1 and 2 are given by:

$$-\frac{\partial}{\partial z}\,(w_1\,N) = \frac{v_1\,N}{r} + \frac{1}{p_g}\,\frac{\partial}{\partial r}\,(v_1\,N\,p_g), \tag{25}$$

$$-\frac{\partial}{\partial z}\,[w_2(1-N)] = \frac{v_2(1-N)}{r} + \frac{1}{p_g}\,\frac{\partial}{\partial r}\,[v_2(1-N)\,p_g] \tag{26}$$

where constant axial pressure, but not radial balanced diffusion has been assumed.

Addition of equations (26) and (25) gives:

$$-\frac{\partial}{\partial z}\,[w_1\,N + w_2(1-N)] = f(r)\,,$$

$$f(r) = \frac{v_1\,N + v_2(1-N)}{r} + \frac{1}{p_g}\,\frac{\partial}{\partial r}\,\{[v_1\,N + v_2(1-N)]\,p_g\} \tag{27}$$

where $f(r)$ is a function of r only.

Integration of equation (27) with respect to z gives:

$$w_1\,N + w_2(1-N) = f(r)\,z + C_3 \tag{28}$$

where C_3 is the integration constant. The left hand side of equation (28) is the mean axial velocity of the gas w_g, since:

$$w_g = w_1\,N + w_2(1-N)\,. \tag{29}$$

Because the gas pressure was assumed to be invariant in the z-direction, w_g must also be independent of z for flow continuity, and hence $f(r) = 0$. This occurs with radial balanced diffusion, as can be seen from equations (27) and (4), with $P_r = 0$.

Substitution of equation (29) in (24) gives:

$$w_1 = w_g - \frac{D_{12}}{N}\,\frac{\partial N}{\partial z}\,. \tag{30}$$

Multiplication by N and differentiation with respect to z gives:

$$-\frac{\partial}{\partial z}\,(w_1\,N) = -w_g\,\frac{\partial N}{\partial z} + D_{12}\,\frac{\partial^2 N}{\partial z^2}\,.$$

Substituting from equations (25), (22) and (11) gives:

$$w_g = w + D_{12}\,\frac{\partial^2 N/\partial z^2}{\partial N/\partial z} \tag{31}$$

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[4] J. Los, Int. Symposium on Isotope Separation, North-Holland Publishing Company, Amsterdam 1957.

[5] W. Groth, Separation of Isotopes; editor: H. London George Newnes Ltd., London 1961.

where w_g and w are the optimum axial velocities with and without axial back diffusion.

Axial back diffusion thus affects the profile, because D_{12} is a function of the pressure and therefore of the radius. The degree of distortion depends on the ratio of the first and second derivates, which are functions of τ_{1z} and P_z. This can be shown as follows: Consider a column in which r_0 is small, and in which the boundary flow beyond r_c has a constant area $\pi(r_b^2 - r_c^2)$; $r_b - r_c$ is the radial width in which the boundary flow occurs. The flow velocities in the boundary for the conditions of radial balanced diffusion and constant axial pressure, can be calculated by the same method as w_g and w_1. The boundary velocities are:

$$w_{gc} = \frac{2\pi r_c v_{1c} N_c}{\pi(r_b^2 - r_c^2)\,\partial N/\partial z} + D_{12}\frac{\partial^2 N/\partial z^2}{\partial N/\partial z}, \tag{32}$$

$$w_{c1} = w_{gc} - \frac{D_{12}}{N}\frac{\partial N}{\partial z} \tag{33}$$

where w_{gc} is the velocity of the gas in the boundary, w_{c1} is the velocity of component 1 in the boundary.

The material transports are given by:

$$\tau_{1z} = \frac{W_{c1} N_c p_{gc}\,\pi(r_b^2 - r_c^2)}{R\,T} + \int_0^{r_c} \frac{2\pi r N p_g w_1}{R\,T}\,dr, \tag{34}$$

$$P_z = \frac{W_{gc}\, p_{gc}\,\pi(r_b^2 - r_c^2)}{R\,T} + \int_0^{r_c} \frac{2\pi r p_g w_g}{R\,T}\,dr. \tag{35}$$

Substitution of equations (30) to (33) in (34) and (35) gives:

$$\tau_{1z}\frac{\partial N}{\partial z} = C_{21} - C_{22}\left(-\frac{\partial N}{\partial z}\right)^2 + C_{23}\frac{\partial^2 N}{\partial z^2}, \qquad P_z\frac{\partial N}{\partial z} = C_{24} + C_{22}\frac{\partial^2 N}{\partial z^2} \tag{36), (37}$$

where $C_{21} = \frac{D}{M}\left[2\pi r_c^2 N_c^2 (1 - N_c)(M_1 - M_2)\frac{\omega^2}{2\,R\,T} - 4\pi \int_{N_0}^{N_c} N(1+\nu)\,dN\right]$,

$$C_{22} = \frac{D}{M}\pi r_b^2, \quad C_{23} = \frac{D}{M}\left[\pi(r_b^2 - r_c^2) N_c + 2\pi \int_0^{r_c} r N\,dr\right],$$

$$C_{24} = \frac{D}{M}\left[2\pi r_c^2 N_c (1 - N_c)(M_1 - M_2)\frac{\omega^2}{2\,R\,T} - 4\pi \int_{N_0}^{N_c} (1+\nu)\,dN\right], \qquad \nu = \frac{1-2N}{2}(M_1 - M_2)\frac{\omega^2 r^2}{4\,R\,T}.$$

It was assumed in the derivation of equations (36) and (37) that the first and second derivatives of N with respect to z do not vary appreciably with the radius.

The second derivative can be eliminated from equations (36) and (37), giving:

$$\left(\frac{\partial N}{\partial z}\right)^2 + \left(\frac{\tau_{1z} - C_{23} P_z}{C_{22}}\right)\frac{\partial N}{\partial z} + \left(\frac{C_{23} C_{24}}{C_{22}} - C_{21}\right)\frac{1}{C_{22}} = 0. \tag{38}$$

Solution of the quadratic equation gives:

$$\frac{\partial N}{\partial z} = -\left(\frac{\tau_{1z} - C_{23} P_z}{2\,C_{22}}\right) \pm \sqrt{\frac{(\tau_{1z} - C_{23} P_z)^2}{4(C_{22})^2} - \frac{C_{23} C_{24}}{(C_{22})^2} + \frac{C_{21}}{C_{22}}}. \tag{39}$$

Comparison of equations (37), (39) and (31) indicates that the optimum profile is dependent on the product streams. The influence will be most pronounced near the outer periphery of the centrifuge.

2.4 Separative power and maximum axial gradient in the presence of back diffusion

The separative power in the presence of axial back diffusion can be calculated by substituting equations (36) and (37) in (2 a).

Comparison of the resulting equations with equation (23 a), in which axial back diffusion is neglected, indicates that:

$$(\delta U'')_{\text{with diffusion}} = (\delta U')_{\text{without diffusion}} - \frac{C_{22}}{N^2(1-N)^2}\left(\frac{\partial N}{\partial z}\right)^2 + \frac{C_{23} - N\,C_{22}}{N^2(1-N)^2}\frac{\partial^2 N}{\partial z^2}$$

$$\text{where } \delta U' = \frac{C_{21} - N\,C_{24}}{N^2(1-N)^2}.$$

Since $C_{23} \cong N\,C_{22}$

$$(\delta U'') \cong (\delta U') - \frac{C_{22}}{N^2(1-N)^2}\left(\frac{\partial N}{\partial z}\right)^2. \tag{40}$$

The maximum axial gradient occurs at total reflux when $\tau_{1z}=0$ and $P_z=0$. $\delta U''$ is then also zero. The maximum axial gradient can be obtained from either equation (40), or equation (39), as

$$\left(\frac{\partial N}{\partial z}\right)_{\max} = \frac{N(1-N)}{\sqrt{2}\, r_b} (M_1 - M_2) \frac{\omega^2 r_c^2}{2RT} . \tag{41}$$

3. Application to thermal diffusion columns

3.1 Optimum profile

For constant total pressure operation in the thermal diffusion column, equation (14) indicates that the values of A and B in equation (5) are:

$$A_T = -N(1-N)\, H\left(\frac{M_1-M_2}{M_1+M_2}\right)\frac{1}{T}\cdot\frac{\partial T}{\partial r}, \tag{42}$$

$$B_T = N(1-N)/D_{12} . \tag{43}$$

The optimum radial concentration gradient is obtained from equations (7) and (42) as

$$\left(\frac{\partial N}{\partial r}\right)_{\mathrm{opt}} = -\frac{N(1-N)}{2} H\left(\frac{M_1-M_2}{M_1+M_2}\right)\frac{1}{T}\frac{\partial T}{\partial r} . \tag{44}$$

The optimum radial velocity for belanced diffusion is obtained from equations (10), (42) and (43) as:

$$(v_1)_{\mathrm{opt.}} = -\frac{D_{12}}{2T}\frac{\partial T}{\partial r} H\left(\frac{M_1-M_2}{M_1+M_2}\right)(1-N) . \tag{45}$$

The temperature distribution in thet column is calculated by considering only heat conduction through the gas since convection currents do not generally contribute appreciably to the heat transfer. For a constant coefficient of heat conductivity in the gas, the temperature distribution is given by:

$$r(\partial^2 T/\partial r^2) + (\partial T/\partial r) = 0 . \tag{46}$$

Integration gives $\partial T/\partial r = C_1/r$ (47)

and $T = C_1 \ln r + C_2$ (48)

where $C_1 = (T_0 - T_c)/\ln(r_0/r_c)$,

$C_2 = T_0 - C_1 \ln r_0$.

The diffusion coefficient varies approximately as the square of the temperature if the pressure is constant[6].

$$D_{12} = C_4 T^2 . \tag{49}$$

The axial profile can be calculated by substituting equations (44), (45), (46), and (49) in equation (11).

$$w\frac{\partial N}{\partial z} = \frac{C_4 C_1^2}{4r^2} H^2\left(\frac{M_1-M_2}{M_1+M_2}\right)^2 N(1-N)(1-2N) . \tag{50}$$

Equation (50) indicates that w is very small (of second order) and varies as the inverse of the radius squared. Cohen[7] has estimated a static region. If a linear variation of the diffusion coefficient with temperature is assumed, then the power of r is reduced to approximately -1.5, and w has a first order magnitude.

3.2 Boundary flows

The relative magnitudes of the axial molecular flows in the separative region and in the boundaries were calculated.

The total flow G_w produced by w is given by:

$$G_w = \int_{r_0}^{r_c} \frac{2\pi w p_g r\, dr}{RT} . \tag{51}$$

From equations (45), (12), (13), (46), (47) and (50) the axial flows are calculated as:

$$G_w \cong 0, \quad G_0 = Q, \quad G_c = -Q \tag{52}$$

where $Q = \dfrac{\pi N(1-N)\, C_1 C_4}{R(\partial N/\partial z)} H\left(\dfrac{M_1-M_2}{M_1+M_2}\right) p_g$.

This indicates that the axial streams generally have the same order of magnitude for all ratios of the diameters and for all values of r_0. It is generally thought that G_0 becomes insignificant in comparison with the other two axial streams when r_0 is small.

3.3 Maximum separative power

The maximum separative power is calculated by introducing equations (43), (44) and (47) in (8) and integrating.

$$\delta U' = \frac{\pi}{2} H^2\left(\frac{M_1-M_2}{M_1+M_2}\right)^2 \frac{p_g D_{12}}{RT^2}\frac{T_0-T_c}{\ln(r_c/r_0)}\ln\frac{T_0}{T_c} . \tag{53}$$

3.4 Separative powers of columns with different flow profiles

a) Static separative region

The separative powers of columns with profiles other than the optimum were also calculated. Firstly, an alternate approach was used for the column in which there is no axial gas-movement in the separative

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[6] Reference [2], p. 248.

[7] Reference [1], p. 132.

region (i. e. $w=0$ and boundary flow). In the separative region the radial flow of each isotope τ_{1r} is a constant and hence from equation (3)

$$N(r\,v_1/T) = C_5 = \tau_{1r}\,R/2\,\pi\,p_g \cong \text{constant}. \tag{54}$$

The radial concentration gradient is given by equation (14). For balanced diffusion

$$\frac{\partial N}{\partial r} = -N\frac{v_1}{D_{12}} - N(1-N)\,H\left(\frac{M_1-M_2}{M_1+M_2}\right)\frac{1}{T}\,\frac{\partial T}{\partial r}. \tag{55}$$

Substituting equations (47), (49) and (54) in equation (55) gives

$$\frac{\partial N}{\partial r} = -\left[\frac{C_5}{C_4} + N(1-N)\,H\left(\frac{M_1-M_2}{M_1+M_2}\right)C_1\right]\frac{1}{r\,T}. \tag{56}$$

The separative $\delta U'''$ is obtained by substituting equations (54) and (56) in (2 b). Integrating (considering N a constant) gives:

$$\delta U''' = -\frac{2\,\pi\,p_g}{R}\,\frac{C_5}{N^2(1-N)^2}\left[\frac{C_5}{C_1\,C_4} + N(1-N)\,H\left(\frac{M_1-M_2}{M_1+M_2}\right)\right]\ln\frac{T_0}{T_c}. \tag{57}$$

The maximum value of equation (57) is

$$\delta U'''_{\max} = \frac{\pi\,p_g}{2\,R}\,\frac{D_{12}}{T^2}\,\frac{T_0-T_c}{\ln(r_c/r_0)}\,H^2\left(\frac{M_1-M_2}{M_1+M_2}\right)^2\left[\ln\frac{T_0}{T_c}\right] \tag{58}$$

and occurs when $$C_5 = -\frac{C_4\,C_1}{2}\,N(1-N)\,H\left(\frac{M_1-M_2}{M_1+M_2}\right). \tag{59}$$

A comparison of equations (58) and (53) indicates that $\delta U_{\max} = \delta U_{\text{opt.}}$ and hence, the change of profile has not affected the maximum separative power that can be attained (when neglecting second order terms):

b) Natural convection axial circulation

The maximum separative power was also estimated for a column in which the axial circulation is induced by natural convection. This is the configuration generally used in practice. The separative power with such a profile appears to be only a small fraction of the maximum attainable (with the optimum profile). Calculations of $\delta U'$ are made complicated by the severe boundary conditions (i. e. $w=0$ and $v_1=0$ at r_0 and r_c). The behaviour of the column was determined graphically, as is shown in Fig. 2.

A natural convection induced axial flow profile (calculated form the NAVIER–STOKES equations) is shown in Fig. 2 a; the ratio of the radii is 20 and $T_0 \cong 2\,T_c$. The radial isotope velocity v_1 was estimated from the continuity equation. The following conditions were used:

1) continuity equation $-w\dfrac{\partial N}{\partial z} \cong N\left(\dfrac{v_1}{r} + \dfrac{\partial v_1}{\partial r}\right)$,

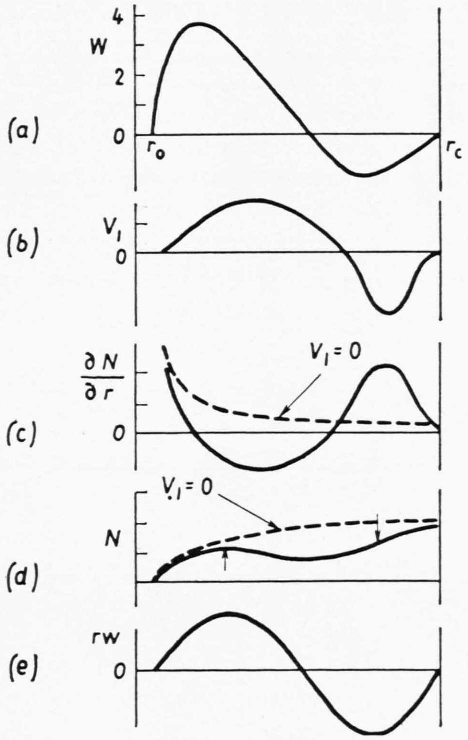

Fig. 2. Radial parameters with natural convection axial circulation. a) Axial gas velocity. b) Radial velocity of isotope component 1. c) Radial concentration gradient of component 1. d) Radial concentration of component 1. e) Axial flow.

2) $(v_1)_{\max}$ occurs when $\partial v_1/\partial r = 0$;

$(v_1)_{\max} = -b\,r\,w$, where $b = \frac{1}{N}\frac{\partial N}{\partial z}$,

3) when $v_1 = 0$; $-b\,w = (\partial v_1/\partial r)$;

$v_1 = 0$ at $r = r_0$ and r_c,

4) when $w = 0$; $v_1 = -r(\partial v_1/\partial r)$.

The radial concentration gradient (Fig. 2 c) and concentration were then plotted using the diffusion equation (55). The effective radii of the upward and downward heavy isotope currents were estimated from Fig. 2 e and inserted as vertical arrows in Fig. 2 d. It was immediately obvious that the difference in concentration between the upward and downward heavy isotope currents is only a small fraction of the total radial change that occurs. The difference in concentration between the axial currents is further a function of the shape of the natural convection profile. These two factors may explain why measurements of collision parameters in columns with natural convection axial circulation are not of high accuracy[8]. A column with a static separative region and boundary flows will be more suitable for this purpose. The results of such a column should be of high accuracy, particularly of the exact equation for the temperature variation of the diffusion coefficient, equation (49), is used. The proposed column will also be a more efficient producer of small amounts of enriched isotopes in the laboratory.

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[8] Reference [3], p. 400.